Magnetic Semiconfinement of Neutral Atoms

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Abstract

A mechanism for creating well-collimated beams of neutral particles or atoms with spins is studied. The consideration is accomplished for a general realistic case, taking into account: (i) the finiteness of a cylindrical trap where the atoms to be shot out, are stored; (ii) the possibility of manipulating the trap magnetic fields in order to form different anisotropic field configurations; (iii) the presence of gravity curving atomic trajectories.

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1 Introduction

Charged particles and ions are known to be easily manipulated by means of electromagnetic fields. The situation is more complicated for neutral particles and atoms. In what follows, we shall often employ, for short, the term "atom" for both elementary neutral particles as well as for composite systems, such as atoms or molecules. Neutral atoms can be successfully confined in different traps (see reviews [1–3]). Recently [4], neutrons were confined in a magnetic trap during the time of about 800 seconds, that is, around 13 minutes; this lifetime being mainly limited by the neutron beta decay into an electron, proton, and anti-neutrino, rather than by the trap losses. The formation of directed beams of neutral atoms is usually done in one of the two following ways. The first way can be called *mechanical*, since it requires the usage of mechanical collimators, selecting atoms from an isotropic distribution, after which the selected particles are released through tubes with a high pressure difference between the ends, as is done in molecular beam masers [5,6]. Another mechanism may be named *resonant*, because it is based on the action of laser beams on resonant atomic transitions [1–3].

A novel mechanism for creating directed beams of neutral atoms has been recently advanced [7,8]. This mechanism can be termed *magnetic*, since it involves magnetic fields, together with a special initial polarization of atomic spins. A short qualitative description of the suggested *magnetic semiconfinement of atoms* is directly connected with its name: This is an effect, when neutral atoms, stored in a trap, begin moving in one direction, forming a well-collimated beam, at the same time being confined from the opposite side.

The ability to form directed well-collimated beams of neutral atoms is very important for various applications. For example, this is necessary for the functioning of atom lasers [2,3]. This can also be used for studying the coherence of trapped Bose-Einstein condensates [3,9], for considering the relative motion of binary mixtures [10], for measuring the one-particle density matrix by the scattering of fast atoms [11], and so on.

In order that a method of creating atomic beams be practical, it is necessary to check its feasibility under a realistic situation. And this is the aim of the present paper to generalize the consideration of the magnetic mechanism for creating atomic beams [7,8] by including the following really existing essential factors.

(i) *Trap shape.* Atoms to be shot out are, first, stored in a trap having a finite size and a particular shape [1–3]. The formation of an atomic beam is accomplished inside the trap whose shape, therefore, is of importance.

(ii) *Field anisotropy.* Fields governing the dynamics of atoms can be prepared in different configurations [1–3,12]. It is useful to include in the consideration additional parameters regulating the anisotropy of field configurations, which would help in choosing optimal conditions for the formation of beams.

(iii) *Influence of gravity.* Atoms, contrary to photons, possess mass. Hence atoms are subject to gravity that may strongly influence their dynamics [3,13]. Therefore one has to accurately take into account the presence of gravity.

2 Semiconfining regime of motion

Let us consider a system of atoms, each having mass m_0, magnetic moment μ_0, and spin S. Introduce the quantum-mechanical averages for the spatial position of an atom $\mathbf{r} = \{r^\alpha\}$, with $\alpha = x, y, z$, and for a spin $\mathbf{S} = \{S^\alpha\}$. Denote the gravitational acceleration as $\mathbf{g} = \{g^\alpha\}$ and the force due to interatomic interactions by $\mathbf{f} = \{f^\alpha\}$. The evolution equations for the averages $\mathbf{r}$ and $\mathbf{S}$, in the semiclassical approach [14,15], can be presented in the form

$$\frac{d^2 r^\alpha}{dt^2} = \frac{\mu_0}{m_0}\, \mathbf{S} \cdot \frac{\partial \mathbf{B}}{\partial r^\alpha} + g^\alpha + \frac{f^\alpha}{m_0}\ , \tag{1}$$

where $\mathbf{B}$ is a magnetic field acting on an atom, and

$$\frac{d\mathbf{S}}{dt} = \frac{\mu_0}{\hbar}\, \mathbf{S} \times \mathbf{B}\ . \tag{2}$$

The total magnetic field of a trap can be written as a sum

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 \tag{3}$$

of a time-invariant trapping field $\mathbf{B}_1$ and an alternating field $\mathbf{B}_2$. As a trapping field, one often uses the quadrupole field

$$\mathbf{B}_1 = B'_1 \left(x\mathbf{e}_x + y\mathbf{e}_y + \lambda z\mathbf{e}_z \right)\ , \tag{4}$$

typical of magnetic traps [1–3], where $\mathbf{e}_\alpha$ are unitary vectors, $B'_1 > 0$ is a field gradient and λ is an anisotropy parameter that can be varied. The alternating field

$$\mathbf{B}_2 = B_2 \left(h_x\mathbf{e}_x + h_y\mathbf{e}_y \right)\ , \tag{5}$$

with $h_\alpha = h_\alpha(t)$ and $h_x^2 + h_y^2 = 1$, is a transerse field serving for the stabilization of atomic confinement.

As is clear from the formulas (4) and (5), these magnetic fields describe a generalized TOP (time orbiting potential) trap. The fields specific for the latter follow if in Eq. (4) one sets $\lambda = -2$ and in Eq. (5) a rotating transverse field is assumed. We accept here a more general form of the trap fields in order to stress that the considered mechanism of beam creation does not compulsory require the usage of the TOP traps but can be realized in other traps as well. Moreover, contrary to the TOP trap, with a fixed anisotropy, there exists a rich variety of other traps, where the field anisotropy can be varied in a wide diapason, with the ratio of radial to axial frequencies reaching a factor of 100. More details on possible field configurations can be found in reviews [1–3]. Modelling here the trap anisotropy by the parameter λ, we demonstrate the main idea that the semiconfining regime of motion can always be optimized by adjusting the corresponding configuration of trap fields.

In what follows, it is convenient to pass to the dimensionless space variable $\mathbf{r} = \{x, y, z\}$ with the dimensionless components

$$x \equiv \frac{r^x}{R_0}\ , \qquad y \equiv \frac{r^y}{R_0}\ , \qquad z \equiv \frac{r^z}{R_0}$$

measured in units of the characteristic length

$$R_0 \equiv \frac{B_2}{B_1'} \; . \tag{6}$$

The latter is called the death circle, which describes the position of the oscillating zero magnetic field. The length (6) limits the size of the trapable cloud whose actual radius is slightly smaller [1–3]. Introduce the characteristic frequency of atomic motion, ω_1, and that of spin motion, ω_2, by the equalities

$$\omega_1^2 \equiv \frac{\mu_0 B_1'}{m_0 R_0} \; , \qquad \omega_2 \equiv \frac{\mu_0 B_2}{\hbar} \; . \tag{7}$$

And let us employ the notation

$$\mathbf{G} \equiv \frac{\mathbf{g}}{R_0 \omega_1^2} \; , \qquad \gamma \vec{\xi} \equiv \frac{\mathbf{f}}{m_0 R_0} \; , \tag{8}$$

where $\mathbf{G}$ means the dimensionless gravitational force and γ implies a collision rate. Atomic collisions can be modelled by a random variable $\vec{\xi} = \{\xi_\alpha(t)\}$ defined by means of the stochastic averages

$$\ll \xi_\alpha(t) \gg = 0 \; , \qquad \ll \xi_\alpha(t) \, \xi_\beta(t') \gg = 2D_\alpha \delta_{\alpha\beta} \, \delta(t - t') \; , \tag{9}$$

in which D_α is a diffusion rate.

With this notation, the evolution equation (1) takes the form of the stochastic differential equation

$$\frac{d^2 \mathbf{r}}{dt^2} = \omega_1^2 \left(S^x \mathbf{e}_x + S^y \mathbf{e}_y + \lambda S^z \mathbf{e}_z + \mathbf{G} \right) + \gamma \xi \; , \tag{10}$$

and the spin equation (2) can be transformed as

$$\frac{d\mathbf{S}}{dt} = \omega_2 \hat{A} \mathbf{S} \; , \tag{11}$$

where $\hat{A} = [A_{\alpha\beta}]$ is an antisymmetric matrix with the elements

$$A_{\alpha\alpha} = 0 \; , \qquad A_{\alpha\beta} = -A_{\beta\alpha} \; ,$$

$$A_{12} = \lambda z \; , \qquad A_{23} = x + h_x \; , \qquad A_{31} = y + h_y \; .$$

The evolution equations can be simplified by taking account of the existence of small parameters

$$\left| \frac{\omega_1}{\omega_2} \right| \ll 1 \; , \qquad \left| \frac{\omega}{\omega_2} \right| \ll 1 \; , \qquad \left| \frac{\gamma}{\omega_2} \right| \ll 1 \; , \tag{12}$$

where the notation

$$\omega \equiv \max_t \left| \frac{d}{dt} \mathbf{h}(t) \right| \tag{13}$$

is used. These inequalities are usually valid for realistic traps [1–3]. Then the evolution equations can be reduced to the consideration of atomic motion on the center manifold

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by employing the scale separation approach [13,16], which involves a generalization of averaging techniques [17] to stochastic equations.

Under conditions (12), the matrix $\hat{A}$ can be treated as a quasi-invariant [13,18] with respect to the fastly oscillating spin $\mathbf{S}$. Then the solution to Eq. (11) can be presented as a sum

$$\mathbf{S}(t) = \sum_{j=1}^{3} C_i \mathbf{b}_j(t) \exp\{\omega_2 a_j(t)t\} \tag{14}$$

over the eigenvectors $\mathbf{b}_j = \mathbf{b}_j(t)$ of the matrix $\hat{A}$, where $a_j = a_j(t)$ are the related eigenvalues, and the coefficients C_j are given by the initial conditions, so that

$$C_j = \mathbf{S}(0) \cdot \mathbf{b}_j(0) \ ,$$

$$\mathbf{b}_j = \frac{(A_{12}A_{23} - a_j A_{31})\mathbf{e}_x + (A_{12}A_{31} + a_j A_{23})\mathbf{e}_y + (A_{12}^2 + a_j^2)\mathbf{e}_z}{(A_{12}^2 - |a_j|^2)^2 + (A_{12}^2 + |a_j|^2)(A_{23}^2 + A_{31}^2)]^{1/2}} \ ,$$

$$a_{1,2} = \pm i\sqrt{A_{12}^2 + A_{23}^2 + A_{31}^2} \ , \qquad a_3 = 0 \ .$$

The guiding center for the spatial variable $\mathbf{r}$ is described by Eq. (10) in which one has to substitute the form (14) and to average the right-hand side according to the rule

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} \ll f(\mathbf{r}, \mathbf{h}(t), \vec{\xi}, t) \gg \ dt \ .$$

In this way, one obtains the equation

$$\frac{d^2\mathbf{r}}{dt^2} = \omega_1^2 (\mathbf{F} + \mathbf{G}) \ , \tag{15}$$

in which

$$\mathbf{F} = C_3 \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} (b_3^x \mathbf{e}_x + b_3^y \mathbf{e}_y + \lambda b_3^z \mathbf{e}_z) \, dt \ ,$$

$$C_3 = \frac{(x + h_x^0)S_0^x + (y + h_y^0)S_0^y + \lambda z S_0^z}{[(x + h_x^0)^2 + (y + h_y^0)^2 + \lambda^2 z^2]^{1/2}} \ ,$$

$$\mathbf{b}_3 = \frac{(x + h_x)\mathbf{e}_x + (y + h_y)\mathbf{e}_y + \lambda z \mathbf{e}_z}{[(x + h_x)^2 + (y + h_y)^2 + \lambda^2 z^2]^{1/2}} \ ,$$

where $h_\alpha^0 \equiv h_\alpha(0)$ and $S_0^\alpha \equiv S^\alpha(0)$. In the integration, defining the effective force $\mathbf{F}$, one should keep in mind the occurrence of another small parameter

$$\left|\frac{\omega_1}{\omega}\right| \ll 1 \ , \tag{16}$$

because of which $\mathbf{r}$ is to be treated as a quasi-invariant with respect to $\mathbf{h}$. Taking, for concreteness, the rotating field

$$h_x(t) = \cos \omega t \ , \qquad h_y(t) = \sin \omega t \tag{17}$$

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results in the effective force

$$\mathbf{F} = \frac{[(1+x)S_0^x + yS_0^y + \lambda zS_0^z](x\mathbf{e}_x + y\mathbf{e}_y + 2\lambda^2 z\mathbf{e}_z)}{2[(1+2x+x^2+y^2+\lambda^2 z^2)(1+x^2+y^2+\lambda^2 z^2)]^{1/2}} \ . \tag{18}$$

We have also considered other forms of the transverse field $\mathbf{h}$, provided it satisfies conditions (12), (13), and (16), which yields slightly different formulas for the effective force $\mathbf{F}$. However, slight variations of $\mathbf{F}$ do not qualitatively change the dynamics of atoms. Therefore in what follows, we particularize the effective force by Eq. (18).

At this point, it is worth stressing that the force (18), caused by magnetic fields, acts on atoms only inside the trap, while outside of the latter the force vanishes. Hence, to consider the motion of atoms in the whole space, including the state when they fly out of the trap, it is necessary to concretize the trap shape. For instance, a cylindrical trap of radius R and length L, measured in units of R_0, can be characterized by one of the shape factors

$$\Xi(\mathbf{r}) = 1 - \Theta\left(x^2 + y^2 - R^2\right) \ \Theta\left(|z| - \frac{L}{2}\right) \ , \qquad \Xi(\mathbf{r}) = \exp\left(-\frac{x^2+y^2}{R^2} - \frac{z^2}{L^2}\right) \ , \tag{19}$$

in which $\Theta(\cdot)$ is a unit step function. Thus, the general expression of the force, acting on atoms inside as well as outside the trap, is Eq. (18) multiplied by a shape factor from Eq. (19). It could be possible to introduce more refined shape factors. However, to study the basic role of the trap finiteness, it is sufficient to consider the simple expressions (19). Generally, the shape factors influence atomic motion only in the intermediate region, at the instant the atoms leave the trap. In this region, the effective force (18) of trapping fields smoothly diminishes to zero. Because of this, an exact form of a shape factor does not change much in the principal behaviour of semiconfined atoms whose main properties remain practically not altered. In our numerical calculations, we have studied the difference resulting from the usage of the step-like and smooth shape factors from Eq. (19). For the same trap parameters R and L, the difference in the atomic trajectories is practically unnoticeable. A slight distinction can only be noticed in the phase portraits for velocities: The step-like shape factor causes the appearance of characteristic kinks that are rounded off by the smooth shape factor.

To complete the formulation of the problem, we have to fix the initial spin polarization $\mathbf{S}_0 \equiv \mathbf{S}(0)$. As is shown in Refs. [7,8], for creating directed motion of atoms, one has to fix the initial spin polarization so that

$$S_0^x = 0 \ , \qquad S_0^y = 0 \ , \qquad S_0^z = S \ . \tag{20}$$

Such initial conditions for spin polarization can be prepared in one of the ways well known in quantum mechanics [19] or by a short resonant pulse, as is discussed in Ref. [20]. Accepting the initial condition (20) and taking account of the shape factor (19), we come to the expression for the effective force

$$\mathbf{F} = \frac{1}{2} \lambda S u z \left(x\mathbf{e}_x + y\mathbf{e}_y + 2\lambda^2 z\mathbf{e}_z\right) \tag{21}$$

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where $u = u(\mathbf{r})$, with

$$u(\mathbf{r}) \equiv \frac{\Xi(\mathbf{r})}{[(1 + 2x + x^2 + y^2 + \lambda^2 z^2)(1 + x^2 + y^2 + \lambda^2 z^2)]^{1/2}} . \tag{22}$$

As is easy to notice, the force (21) is invariant under the transformation

$$\lambda \to -\lambda , \qquad S \to -S . \tag{23}$$

Therefore, it is always possible to choose such λ and S that $\lambda S > 0$, which is assumed in what follows.

It is convenient to pass to the dimensionless time measured in units of $(\sqrt{\lambda S}\, \omega_1)^{-1}$. The return to the dimensional time is made by means of the replacement $t \to \sqrt{\lambda S}\, \omega_1 t$. Also, let us introduce the dimensionless gravitational force

$$\vec{\delta} \equiv \frac{\mathbf{G}}{\lambda S} = \frac{\mathbf{g}}{\lambda S R_0 \omega_1^2} = \{\delta_\alpha\} . \tag{24}$$

Then the evolution equation (15) can be written as the system of equations

$$\frac{d^2 x}{dt^2} = \frac{1}{2}\, uzx + \delta_x , \qquad \frac{d^2 y}{dt^2} = \frac{1}{2}\, uzy + \delta_y ,$$

$$\frac{d^2 z}{dt^2} = \lambda^2 uz^2 + \delta_z , \tag{25}$$

where the function $u = u(\mathbf{r})$ is defined in Eq. (22), with the shape factor (19).

We accomplished numerical solution of Eqs. (25) for different field-anisotropy parameters λ and for different trap radii R and lengths L. The results are presented in Figs. 1 to 4, where the z axis is along the trap axis and the latter is inclined by the 45 degrees to the horizon, so that $\delta_z = -\delta_x$. Initial conditions for the spatial position of atoms are close to the trap center $\mathbf{r}_0 = 0$, and initial velocities of atoms, $\dot{\mathbf{r}}_0$, are varied in the interval $-0.1 \leq \dot{r}_0^\alpha \leq 0.1$. The motion $x(t)$ is similar to $y(t)$, because of which we show only the $x - z$ cross-section of phase portraits. The pictures do not qualitatively change when varying the trap parameters R and L in the interval $[1, 10]$. But the acceleration is better for longer traps, with larger L, as well as for larger λ. The maximal velocity that can be achieved is of the order of $v_{max} \sim \lambda\sqrt{L}$. The aspect ratio $R_{asp} = <x> / <z>$ is small, $R_{asp} \sim 0.01$, showing that the beam is stretched in the z-direction about 100 times larger than in the x-direction, that is, the beam is well collimated. The figures present calculations for the step-like shape factor which, to our mind, more clearly illustrates the role of the trap finiteness. As we have numerically checked, the usage of the smooth factor, with the same trap parameters, does not alter atomic trajectories. The sole difference appears in the phase portraits showing the axial to radial velocities: In the case of the step-like shape factor there are kinks in the intermediate region where atoms leave the trap. These kinks are rounded off in the case of the smooth shape factor.

Comparing these numerical calculations with those of Ref. [8], where gravity was not taken into account, we come to the following conclusions. The regime of motion, called

semiconfinement, remains, when atoms are confined from one side of the trap by the minimal value

$$z_{min} \approx - \left(\frac{6}{\lambda^2} \, \dot{z}_0^2 \right)^{1/3} \, ,$$

but are not confined from another side, escaping with acceleration in this direction. For the present choice of coordinates, the escape direction is along the axis z. If the gravity component $\delta_z > 0$, the positive gravitational force only helps to atoms to move preferably in the z-direction, so that all atoms are semiconfined and escape from the trap forming a narrow beam along the z-axis. However, if $\delta_z < 0$, then gravity hampers to escape for some of the atoms. The fraction of atoms that remain confined because of gravity is of order $\frac{1}{2}\lambda\sqrt{\frac{1}{3}|\delta_z|}$. The main visible influence of gravity on the ejected beam is in curving atomic trajectories in the same way as gravity curves the trajectory of a ball shot out of a cannon.

To have a feeling of the values of the characteristic quantities, met above, let us give estimates for the case of parameters typical of alkali atoms in magnetic traps [1–3,9,21]. The characteristic frequency of atomic motion is $\omega_1 \sim 10^2 - 10^3$ s^{-1}, that of spin motion is $\omega_2 \sim 10^7 - 10^8$ s^{-1}. The frequency of the transverse rotating field is $\omega \sim 10^4 - 10^5$ s^{-1}. The collision rate is $\gamma \sim 10$ s^{-1}. Consequently,

$$\frac{\omega_1}{\omega_2} \sim 10^{-5} \, , \qquad \frac{\omega}{\omega_2} \sim 10^{-3} \, , \qquad \frac{\gamma}{\omega_2} \sim 10^{-6} \, , \qquad \frac{\omega_1}{\omega} \sim 10^{-2} \, ,$$

from where it follows that the inequalities (12) and (16) hold true. The characteristic radius (6) is $R_0 \sim 0.1 - 0.5$ cm. The dimensionless gravity components, defined in Eq. (24), can always be made small. Thus, for $\lambda \sim 2$, $S \sim 1$, $R_0 \sim 0.5$ cm, and $\omega_1 \sim 10^2 - 10^3$ s^{-1}, substituting in Eq. (24) the gravitational acceleration $g^\alpha \sim 10^3$ cm/s^2, we get $\delta_\alpha \sim 10^{-3} - 10^{-1}$. When the trap axis is directed against the gravitational force, then the amount of atoms that cannot escape from the trap is between 1% to 10%. Hence, the semiconfining regime of motion is not much spoiled by gravity. The majority of atoms fly out of the trap, forming a well-collimated beam with a squeezing factor of about 100; the maximal velocity of atoms, for a trap of length $1 - 10$ cm can reach the order of 100 cm/s.

The role of random pair collisions can be considered in the same way as in Ref. [8]. These collisions do not essentially disturb the semiconfining regime provided that

$$\frac{\gamma^2 D}{(\lambda S)^{3/2}\omega_1^3} \ll 1 \, , \qquad D \equiv \sup_\alpha\{D_\alpha\} \, .$$

This condition, accepting for the diffusion rate an estimate $D \sim k_B T/\hbar$, where k_B is the Boltzmann constant and T is temperature, can be presented as

$$T \ll T_c \equiv \frac{\hbar(\lambda S)^{3/2}\omega_1^3}{k_B \gamma^2} \, .$$

These inequalities show that the disorganizing influence of atomic collisions is suppressed when increasing the anisotropy parameter λ or lowering temperature. For the characteristic values $\lambda S \sim 1$, $\omega_1 \sim 10^2 - 10^3$ s^{-1}, and $\gamma \sim 10$ s^{-1}, the critical temperature,

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below which the semiconfining regime appears, is $T_c \sim 10^{-7} - 10^{-4}$ K. This implies that for successful functioning of an atom cannon, trapped atoms are to be sufficiently cooled down. Such a cooling can be effectively realized in modern traps [1–3].

3   Discussion

In this paper, we have analysed the influence on the magnetic semiconfinement of neutral atoms of three factors, trap size, field anisotropy, and gravity, which had not been considered in the previous works [7,8]. The principal result is that, despite the action of gravity, the semiconfining regime can always be preserved by appropriately choosing trap parameters. Consequently, the suggested magnetic semiconfinement can really serve as an efficient mechanism for creating well-collimated beams of neutral particles and atoms.

In order to correctly describe the semiconfining regime of motion, it has been necessary to lift the adiabatic approximation that is usually employed for considering the motion of trapped atoms. As is known from the general theory [17], the adiabatic approximation may be reasonable for treating the motion of a multifrequency dynamical system that is close to a stationary state. The motion of atoms, permanently confined inside a trap, is exactly such a kind of a stationary regime. This is why the adiabatic approximation works reasonably well for constantly confined atoms. Corrections to this approximation, in the case of a TOP trap, can be estimated as being of the order of $\omega_1/\omega \sim 10^{-2}$, which being expressed in percent, makes at most a few percent. Atomic micromotion of atoms in a TOP trap has recently been studied experimentally [22,23]. The detected nonadiabatic effects were found to be of the order of a few percent, in agreement with the above estimate. Anharmonic corrections to the harmonic approximation of the effective TOP potential are also not large for trapped atoms moving close to the trap center [24].

The situation is different for atoms accelerated out of trap in the semiconfining regime of motion. Such a regime is far from stationary, because of which the adiabatic approximation is not applicable in principle [17]. Also, in this regime, atoms may move far from the trap center, which prohibits the usage of a harmonic approximation. This is why, we have considered the effective trap force (21) in its complete form, containing the factor (22), never invoking harmonic approximations. At the same time, taking into account the anharmonic factor (22) not only makes the consideration more general but also reduces the influence of a particular shape factor on the motion of atoms. This happens because the role of the shape factor becomes noticeable when atoms leave the trap. But the effective force (21), due to the anharmonic factor (22), fastly diminishes to zero after atoms leave the death circle whose radius is usually essentially smaller than the trap size.

As is explained in the last paragraph of the previous section, the semiconfining regime is easier realized after precooling to sufficiently low temperatures. In the region of the estimated temperatures, Bose-Einstein condensates can be created. Atomic beams outcoupled from condensates remind photon beams from lasers, because of which a device emitting condensed atoms can be called an atom laser. The known standard way for realizing an output coupler for trapped atoms is by transferring atoms from a trapped state into an untrapped state by means of a monochromatic resonant rf field, as has been done

in experiments [25–27]. This method produces a beam of atoms merely falling down in the field of gravity. Such a device can hardly be called an atom laser, since the very first condition on a laser is that its output could be pointed in an arbitrary direction. A directional extraction of sodium atoms from a trapped condensate was demonstrated by using two laser beams stimulating Raman transitions [28]. More information on atom lasers can be found in reviews [2,3]. The mechanism for creating atomic beams, we suggest, differs from the mentioned above in three main points: (i) it can be realized by using only magnetic fields, without involving additional blocking devices or external laser beams; (ii) the beam direction is prescribed by the trap axis that can be oriented arbitrarily; (iii) the characteristics of the semiconfinement can be varied by choosing appropriate magnetic fields. In this way, the proposed mechanism can be employed for creating highly-directional beams from atom lasers.

It would, certainly, be nice if the effect of semiconfinement could be observed experimentally. Since we are not experimentalists, it would not be reasonable from our side to plunge into a discussion of technical details of experiments. Our aim has been to demonstrate the principal feasibility of realizing the semiconfining regime of motion. However, to motivate experimentalists, it would, probably, be useful to touch several points whose clarification could facilitate experimental work.

First of all, we would like to emphasize that the described effect is rather general and can be realized in different traps, with different magnetic fields. A dynamic TOP-like trap was considered above for concreteness. As we have checked, the semiconfining regime of motion exists for static traps as well, provided the required initial conditions for atomic spins are prepared. We could imagine at least three ways to achieve the necessary initial spin polarization.

One possibility could be to orient spins in the z-direction by means of a guiding longitudinal field, while the transverse field is yet switched off. After this, the latter field should be quickly switched on. The main problem here is that the trap field has to be switched on faster than the Larmor period $2\pi/\omega_2$. This looks to be difficult to accomplish experimentally. Although, what seems difficult today may be successfully performed in future.

Another possibility is to prepare, first, atoms with the required z-polarization of spins in a trap. For instance, this can be done by using the trap of Ref. [29], which is a quadrupole trap with a bias field along the z-axis. Then the spin-polarized atoms are to be quickly loaded into another trap with the desired field configuration. The feasibility of transferring atoms from one trap to another by means of a sudden transfer, as opposed to slow transfer, has been discussed in Ref. [30].

One more way could be as follows. Assume that a trap consists of two chambers, upper and lower. Let atoms be confined in the upper chamber. Then, acting on these atoms by rf field, one transfers them into an untrapped state [25–28]. Obtaining the spin polarization corresponding to this untrapped state, atoms fall down because of gravity, and pass to the lower chamber. The field configuration of the latter is to be such that the spin polarization of the falling atoms be that required for the semiconfining regime of motion. Thus, the lower chamber could emit a quasi-continuous beam of atoms.

The features of the emitted atomic beam depend, of course, on many technical details

related to particular experiments. The major properties are prescribed by the magnetic field configuration arranged. The trap shape factor, as is explained above, plays less important role, when the trap size is larger than the radius of the death circle or the radius of an atomic cloud in a trap. A continuous atomic beam being ejected out of the trap will be influenced by the trap shape slightly stronger than a short pulse. In the latter case, when the group of atoms, after an initial acceleration reaches the trap boundary, one could switch off the trap field abruptly. Such a situation would exactly correspond to the usage of the step-like shape factor.

Finally, one could imagine experiments for which the directed motion of atoms, organized in the proposed way, would not necessarily end up with atoms leaving the trap. Some of such possibilities are mentioned in the Introduction. Probably, the most interesting suggestion would be to study the relative motion of one atomic component through another, as is discussed in Ref. [7].

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Figure Captions

Fig. 1. The trajectories of atoms in the $x - z$ cross-section at the initial stage of acceleration, when $0 \leq t \leq 50$, for the trap parameters $R = 1$, $L = 1$, $\lambda = 20$, and the gravity components $\delta_x = 0.01$, $\delta_y = 0$, $\delta_z = -0.01$.

Fig. 2. The velocities of atoms on the $\dot{x} - \dot{z}$ plane for the same conditions as in Fig. 1. The characteristic kinks is a result of a step-like shape.

Fig. 3. Typical trajectories during the long time interval $0 \leq t \leq 300$ for the trap parameters $R = 1$, $L = 1$, $\lambda = 10$, and the gravity components $\delta_x = 0.05$, $\delta_y = 0$, $\delta_z = -0.05$.

Fig. 4. The phase portrait for atomic velocities corresponding to the parameters of Fig. 3. The kinks are again due to a step-like shape factor. The general picture for a smooth shape factor remains the same, except that the kinks become rounded off.